Exercise 2.8.2

Sketch the slope field for the following differential equations. Then "integrate" the equation manually by drawing trajectories that are everywhere parallel to the local slope.

a) $\dot{x} = x$ b) $\dot{x} = 1 - x^2$ c) $\dot{x} = 1 - 4x(1 - x)$ d) $\dot{x} = \sin x$

Solution

Each of the ODEs can be written as

$$\dot{x} = f(x).$$

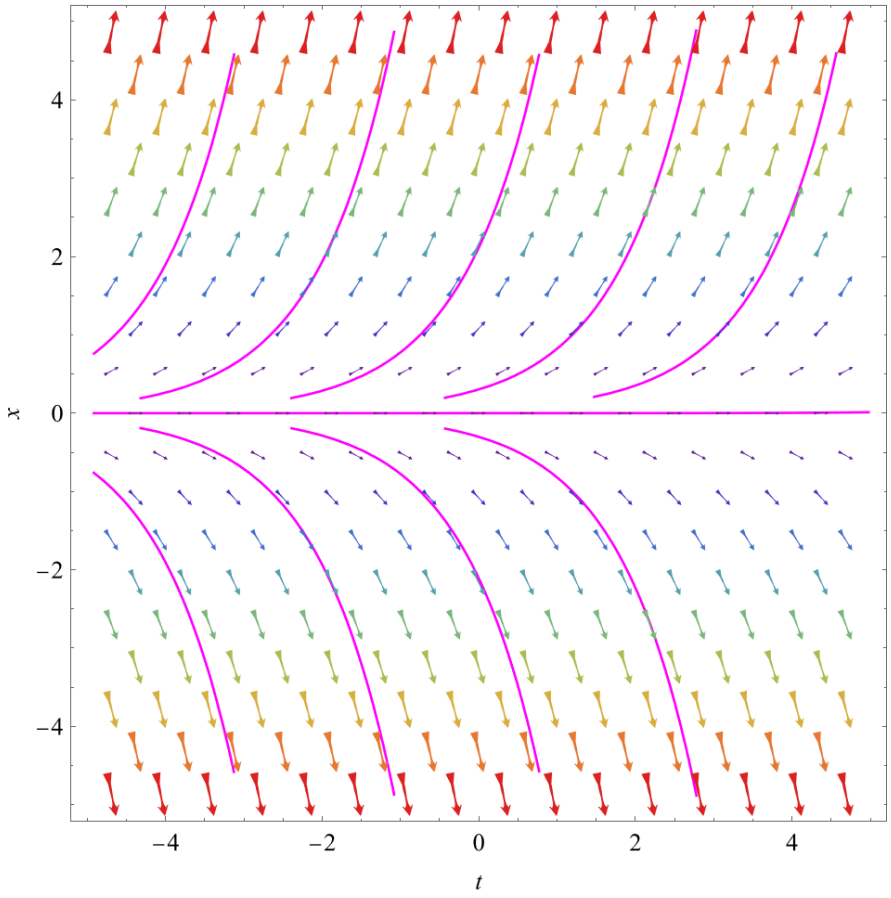
The slope field is obtained by writing

$$\langle dt, dx \rangle = \left\langle 1, \frac{dx}{dt} \right\rangle dt = \langle 1, \dot{x} \rangle \, dt = \langle 1, f(x) \rangle \, dt$$

and plotting the vector field $\langle 1, f(x) \rangle$ in the tx-plane. The (magenta) streamlines are possible solutions to the differential equation.

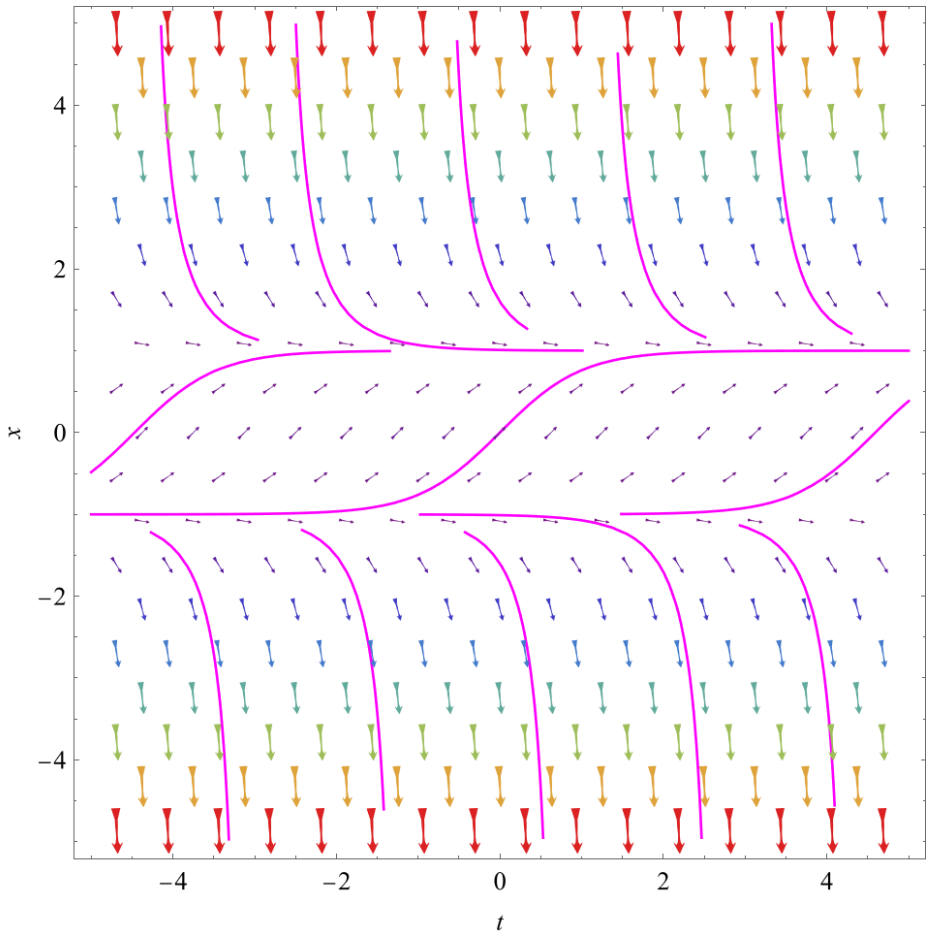
Part (a)

The ODE here is $\dot{x} = x$, so the vector field plotted is $\langle 1, x \rangle$.



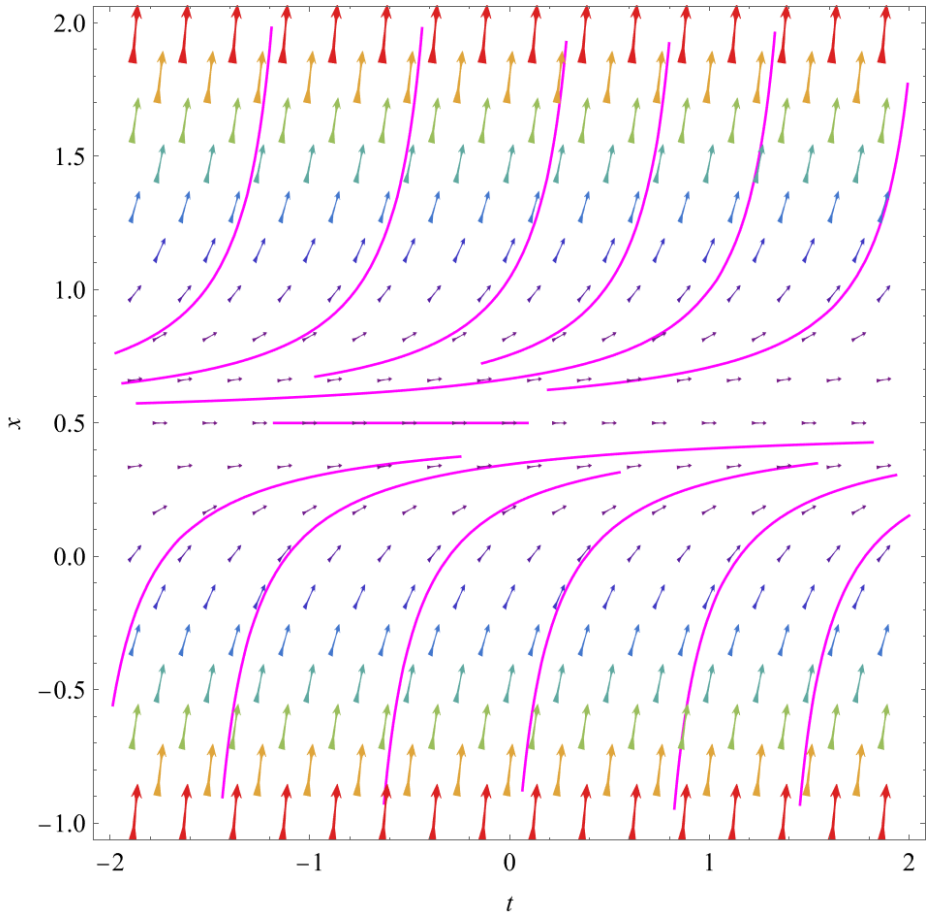
Part (b)

The ODE here is $\dot{x} = 1 - x^2$, so the vector field plotted is $\langle 1, 1 - x^2 \rangle$.



Part (c)

The ODE here is $\dot{x} = 1 - 4x(1-x)$, so the vector field plotted is $\langle 1, 1 - 4x(1-x) \rangle$.



Part (d)

The ODE here is $\dot{x} = \sin x$, so the vector field plotted is $\langle 1, \sin x \rangle$.